Delta Robot

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1 Introduction

This exercise consists in simulating the behaviour of a Delta Robot. It aims to introduce, on a simplified model, all the necessary concepts for the modelling, namely:

- Multibody structure with rotation articulations
- Closed-loop kinematics
- Driven joints
- Inverse kinematics
- Inverse dynamics

2 Multibody model

- There are 16 bodies: 3 up legs, 3 middle legs, 3 left down legs, 3 right down legs, 3 platform legs and 1 platform (Fig. 1).
- The bodies are interconnected with each other by different ”elementary” joints with 1 degree of freedom (prismatic or rotoid).
- The relative motion of the *upLeg1* with respect to the base can take place in rotation about the Y axis ($\hat{x}_y^0$)
- Between the upper legs 2 and 3 and the base there is a blocked rotation on the Z axis ($\hat{x}_z^0$)
- The relative motion of the middle legs (*midLeg*) with respect to its respective *upLeg* can take place in rotation about the Y local axis.
- Every down leg (right and left) has a relative rotational motion with respect to its respective middle leg.
- The relative motion of the platform legs (*platLeg*) with respect to each left down leg can take place in 3 directions of rotation. To deal with system kinematic loops, *Ball* cuts (for more information, see RobotranTutorial) will be added between the platform legs and the left down legs.
- The platform of the robot has 3 translational motion coordinates ($\hat{x}_x^0$, $\hat{x}_y^0$, $\hat{x}_z^0$) with respect to the base and 2 relative rotational motion coordinates (y & z) regarding every platform leg.
- The 3 translation motion of the platform are driven (*Forced-Driven*) and defined by the user.

- The 6 relative rotation motion coordinates of the platform are also blocked (*Forced-Driven*) to, this way, define its fixed rotation value and to obtain its equivalent joint forces.
- All these bodies are also subject to gravity $(9,81m/s^2)$.
- The system has 0 degrees of freedom overall:
  - 23 joints (+23)
  - 11 driven joints (-11)
  - 3 user constraints (-3)
  - 3 ball cuts, with 3 algebraic constraints every one (-3x3=-9)

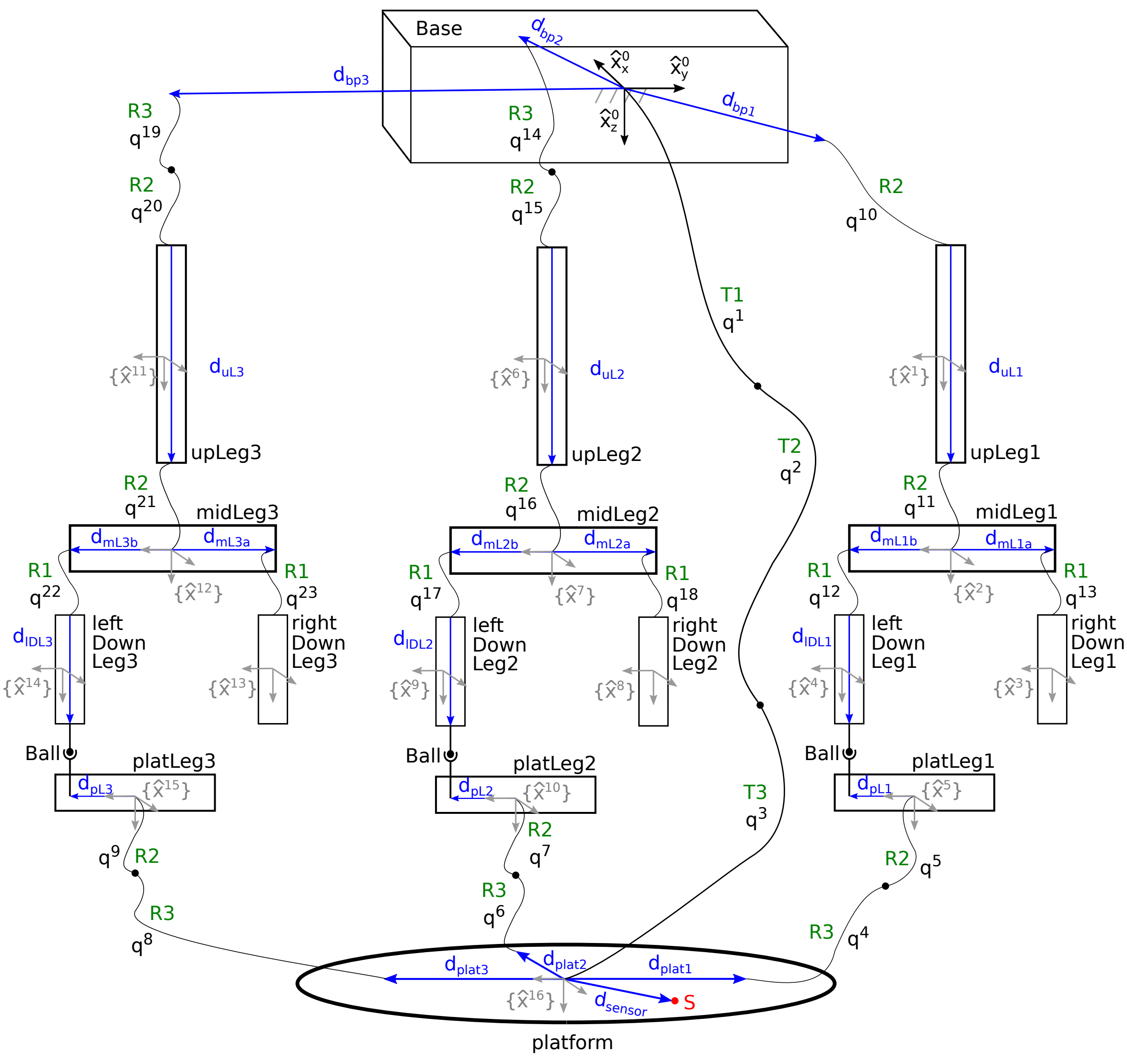

Figure 1: Multibody model of the Delta Robot

3 Robot trajectory definition

- T1 and T2 translational coordinates driven motion definition (*user_DrivenJoints*):

$$q_1 = a \cdot sin(\omega_a \cdot t) \tag{1}$$

$$\dot{q}_1 = a \cdot \omega_a \cdot cos(\omega_a \cdot t) \tag{2}$$

$$\ddot{q}_1 = -a \cdot \omega_a^2 \cdot sin(\omega_a \cdot t) \tag{3}$$

$$q_2 = a \cdot cos(\omega_a \cdot t) \tag{4}$$

$$\dot{q}_2 = -a \cdot \omega_a \cdot sin(\omega_a \cdot t) \tag{5}$$

$$\ddot{q}_2 = -a \cdot \omega_a^2 \cdot cos(\omega_a \cdot t) \tag{6}$$

  where:

  - a = 0.1 m
  - f_a = 1 Hz
  - $\omega_a = 2\pi f_a$

- T3 translational coordinates driven motion definition (*user_DrivenJoints*):

$$q_3 = 0.9 + b \cdot sin(\omega_b \cdot t) \tag{7}$$

$$\dot{q}_3 = b \cdot \omega_b \cdot cos(\omega_b \cdot t) \tag{8}$$

$$\dot{q}_3 = -b \cdot \omega_b^2 \cdot sin(\omega_b \cdot t) \tag{9}$$

  where:

  - b = 0.2 m
  - f_b = 0.5 Hz
  - $\omega_a = 2\pi f_b$

4 Joints friction

- For every rotational coordinate (*user_JointForces*): $\mu = 10$
- Joint forces: $\dot{Q}_{4-23} = -\mu \cdot \dot{q}_{4-23}$

5 User constraints

- For every middle leg, its joint coordinates between every down leg must be equal:
  - $q_{12} = q_{13}$
  - $q_{17} = q_{18}$
  - $q_{22} = q_{23}$

6 Data

- Base:
  - Gravity: g $= [\hat{x}^0]^T \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}; g = 9.81\ m/s^2$
  - Anchor point 1 position: $d_{bp1} = [\hat{x}^0]^T \begin{pmatrix} -0.5\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
  - Anchor point 2 position: $d_{bp2} = [\hat{x}^0]^T \begin{pmatrix} -0.25\ m \\ 0.433\ m \\ 0\ m \end{pmatrix}$
  - Anchor point 3 position: $d_{bp3} = [\hat{x}^0]^T \begin{pmatrix} -0.25\ m \\ -0.433\ m \\ 0\ m \end{pmatrix}$
- Joint Base-upper legs:
  - Base-upLeg1:
    - R2. Angle = 1.0 rad. Nature: *Dependent*.
  - Base-upLeg2:
    - R2. Angle = 1.0 rad. Nature: *Dependent*.
    - R3. Angle = 2.0944 rad. Nature: *Forced-Driven*.
  - Base-upLeg2:
    - R2. Angle = 1.0 rad. Nature: *Dependent*.
    - R3. Angle = -2.0944 rad. Nature: *Forced-Driven*.
- Carrier body:
  - Centre of mass position of each one: $\Big|\text{COM}\Big|_{1,6,11} = [\hat{x}^{1,6,11}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
  - Anchor point 1 (*toDownLeg1*): $d_{uL1} = [\hat{x}^1]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0.5\ m \end{pmatrix}$
  - Anchor point 2 (*toDownLeg2*): $d_{uL2} = [\hat{x}^6]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0.5\ m \end{pmatrix}$
  - Anchor point 3 (*toDownLeg3*): $d_{uL3} = [\hat{x}^{11}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0.5\ m \end{pmatrix}$
- Joint Base-upper legs:
  - Joint type: R2
  - Nature: *Dependent*

- Middle legs:
  - Centre of mass position of each one: $\Big|\text{COM}\Big|_{2,7,12} = [\hat{x}^{2,7,12}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
  - *midLeg1* $\rightarrow$ anchor point *to_RightDL1*: $d_{mL1a} = [\hat{x}^2]^T \begin{pmatrix} 0\ m \\ 0.05\ m \\ 0\ m \end{pmatrix}$
  - *midLeg1* $\rightarrow$ anchor point *to_LeftDL1*: $d_{mL1b} = [\hat{x}^2]^T \begin{pmatrix} 0\ m \\ -0.05\ m \\ 0\ m \end{pmatrix}$
  - *midLeg2* $\rightarrow$ anchor point *to_RightDL2*: $d_{mL2a} = [\hat{x}^7]^T \begin{pmatrix} 0\ m \\ 0.05\ m \\ 0\ m \end{pmatrix}$
  - *midLeg2* $\rightarrow$ anchor point *to_LeftDL2*: $d_{mL2b} = [\hat{x}^7]^T \begin{pmatrix} 0\ m \\ -0.05\ m \\ 0\ m \end{pmatrix}$
  - *midLeg3* $\rightarrow$ anchor point *to_RightDL3*: $d_{mL3a} = [\hat{x}^{12}]^T \begin{pmatrix} 0\ m \\ 0.05\ m \\ 0\ m \end{pmatrix}$
  - *midLeg3* $\rightarrow$ anchor point *to_LeftDL3*: $d_{mL3b} = [\hat{x}^{12}]^T \begin{pmatrix} 0\ m \\ -0.05\ m \\ 0\ m \end{pmatrix}$
- Joint between the middle legs and the down legs:
  - Joint type: R1
  - Nature: *Dependent*
- Right down legs:
  - Centre of mass position of each one: $\Big|\text{COM}\Big|_{3,8,13} = [\hat{x}^{3,8,13}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
- Left down legs:
  - Centre of mass position of each one: $\Big|\text{COM}\Big|_{4,9,14} = [\hat{x}^{4,9,14}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
  - *leftDownLeg1* $\rightarrow$ anchor point *ballLeg1*: $d_{lDL1} = [\hat{x}^4]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 1\ m \end{pmatrix}$
  - *leftDownLeg2* $\rightarrow$ anchor point *ballLeg2*: $d_{lDL2} = [\hat{x}^9]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 1\ m \end{pmatrix}$
  - *leftDownLeg3* $\rightarrow$ anchor point *ballLeg3*: $d_{lDL3} = [\hat{x}^{14}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 1\ m \end{pmatrix}$

- Cuts between the *leftDownLeg* 1-3 and the *platLeg* 1-3
  - Ball cut 1 between the *leftDownLeg1* and the *platLeg1*.
  - Ball cut 2 between the *leftDownLeg2* and the *platLeg2*.
  - Ball cut 3 between the *leftDownLeg3* and the *platLeg3*.
- Platform legs:
  - Centre of mass position of each one: $\Big|\text{COM}\Big|_{5,10,15} = [\hat{x}^{5,10,15}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
  - *platLeg1* $\rightarrow$ anchor point *to_leftDL1*: $d_{pL1} = [\hat{x}^5]^T \begin{pmatrix} 0\ m \\ -0.05\ m \\ 0\ m \end{pmatrix}$
  - *platLeg2* $\rightarrow$ anchor point *to_leftDL2*: $d_{pL2} = [\hat{x}^{10}]^T \begin{pmatrix} 0\ m \\ -0.05\ m \\ 0\ m \end{pmatrix}$
  - *platLeg3* $\rightarrow$ anchor point *to_leftDL3*: $d_{pL3} = [\hat{x}^{15}]^T \begin{pmatrix} 0\ m \\ -0.05\ m \\ 0\ m \end{pmatrix}$
- Joint between the platform legs and the platform
  - Platform - *platLeg1*:
    - R2. Angle = 0 rad. Nature: *Forced-Driven*.
    - R3. Angle = 0 rad. Nature: *Forced-Driven*.
  - Platform - *platLeg2*:
    - R2. Angle = 0 rad. Nature: *Forced-Driven*.
    - R3. Angle = 2.0944 rad. Nature: *Forced-Driven*.
  - Platform - *platLeg3*:
    - R2. Angle = 0 rad. Nature: *Forced-Driven*.
    - R3. Angle = -2.0944 rad. Nature: *Forced-Driven*.
- Platform:
  - Mass: 1 kg
  - Centre of mass position of each one: $\Big|\text{COM}\Big|_{16} = [\hat{x}^{16}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
  - Inertia: $I^6_{xx} = 1.0\ kg \cdot m^2$; $I^6_{yy} = 1.0\ kg \cdot m^2$; $I^6_{zz} = 1.0\ kg \cdot m^2$
  - Anchor point *ballLeg1*: $d_{plat1} = [\hat{x}^{16}]^T \begin{pmatrix} 0.05\ m \\ 0\ m \\ 0\ m \end{pmatrix}$
  - Anchor point *ballLeg2*: $d_{plat2} = [\hat{x}^{16}]^T \begin{pmatrix} -0.025\ m \\ 0.0433\ m \\ 0\ m \end{pmatrix}$

- Anchor point *ballLeg3*: $d_{plat3} = [\hat{x}^{16}]^T \begin{pmatrix} -0.025\ m \\ -0.0433\ m \\ 0\ m \end{pmatrix}$
- Anchor point *sensor*: $d_{sensor} = [\hat{x}^{16}]^T \begin{pmatrix} 0\ m \\ 0\ m \\ 0.1\ m \end{pmatrix}$

7 Objectives

The expected results are:

- Kinematics: time history of the upper legs positioning (Fig. 2) and the platform trajectory (Fig. 3).

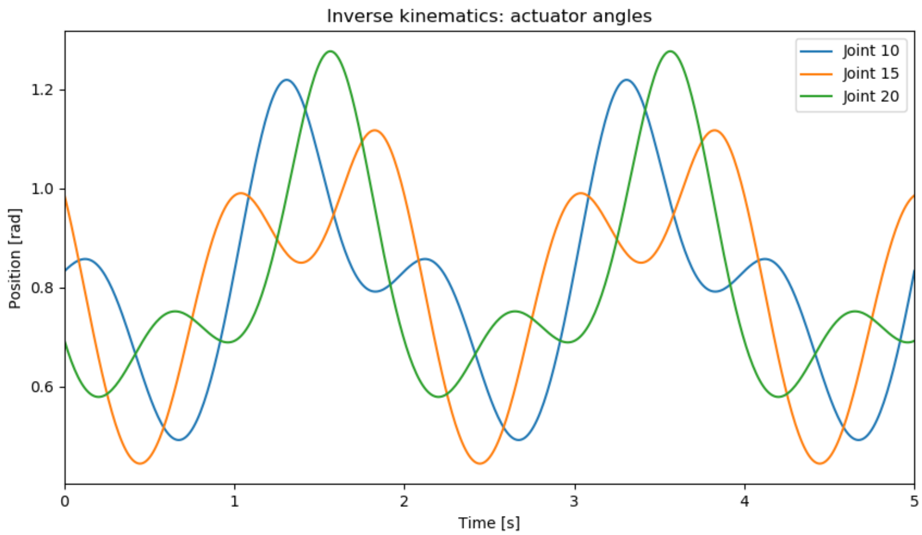

Figure 2: Actuator angles positioning

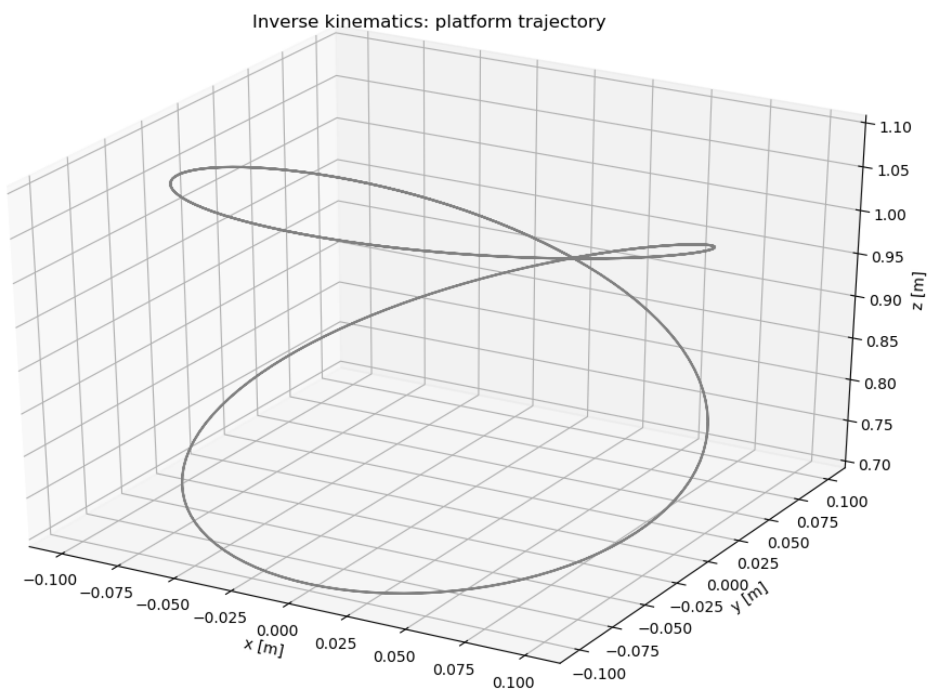

Figure 3: Platform trajectory

- Dynamics: time history of upper legs joint torques (Fig. 4).

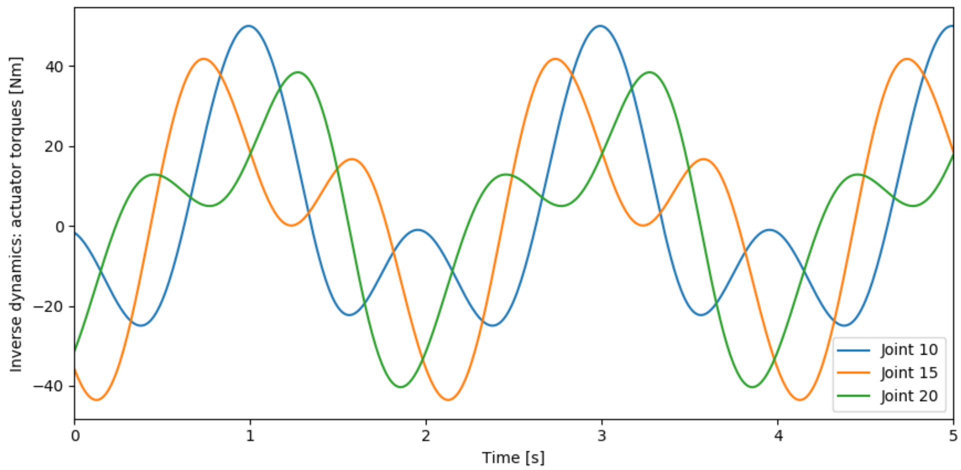

Figure 4: Upper legs-Base joint torques on Z axis